

Dynamical SUSY and R-Symmetry Breaking in SQCD with Massive and Massless Flavors

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We show that supersymmetry and R-symmetry can be dynamically broken in a long-lived metastable vacuum of SQCD with massive and massless flavors. The vacuum results from a competition of a (leading) two-loop effect and small “Planck” suppressed higher-dimension operators. This mechanism provides a particularly simple realization of dynamical SUSY and R-symmetry breaking, and as such it is a good starting point for building phenomenologically viable models of gauge mediation. We take a preliminary step in this direction by constructing a complete model of minimal gauge mediation. Here we find that the parameters of the model are surprisingly constrained by the hidden sector. Similar mechanisms for creating long-lived states operate in a large class of models.

1. Introduction

Dynamical supersymmetry breaking (DSB) via strong gauge dynamics has long been an attractive mechanism for explaining the large hierarchy between the weak scale and the Planck scale [1]. Early, seminal models of DSB were important “existence proofs” – however, they tended to be rather complicated (see e.g. [2-5] for reviews and references). This was tied to the assumption that the DSB vacuum had to be the global minimum of the potential. By relaxing this assumption and allowing for metastability, much simpler models of DSB have recently been constructed, starting with the discovery of metastable vacua in SQCD with massive flavors [6]. (Of course, metastability and its connection to SUSY breaking are not new ideas; for earlier works see e.g. [7-11].) This work has opened new model building avenues, especially for models of direct gauge mediation. Since metastable SUSY-breaking models can be vector-like, they can naturally have large flavor symmetries. This makes them more amenable for direct gauge mediation, where a subgroup of the flavor symmetry is gauged and identified with the SM gauge group [12].

Unfortunately, despite all its positive features, the metastable vacuum of SQCD with massive flavors has one major phenomenological defect: it preserves a continuous $U(1)_R$ symmetry. Without breaking this symmetry somehow, the gauginos of the visible sector would remain massless.¹

Various approaches have been tried to get around this problem. Motivated by the fact that the low-energy effective theory of massive SQCD is simply a generalized O’Raifeartaigh model, more sophisticated O’Raifeartaigh models with R-charge assignments different from 0 or 2 were studied in [14], and it was shown that such models could lead to spontaneous R-symmetry breaking. A detailed study of the phenomenology of such models was undertaken in [15]; however, no UV complete dynamical realization of these models is known. (Non-renormalizable realizations based on massive SQCD with baryonic deformations were constructed and studied in [16,17].) Alternatively, one can gauge an auxiliary $U(1)$ symmetry of the problem; the competition between the gauge and Yukawa couplings can lead to a spontaneous R-symmetry breaking minimum of the

¹ Two comments. First, the R-symmetry is broken explicitly by nonperturbative effects but this is generally too small to allow for realistic gaugino masses. Second, we are assuming here standard Majorana gaugino masses. An interesting alternative whose phenomenology has recently been revisited in [13] is to have Dirac gaugino masses, in which case the R-symmetry need not be broken at all.

one-loop effective potential [18-20]. However, here there generally has to be some fine tuning of the couplings in order to achieve the desired R-symmetry breaking [21]. Finally, a third (and perhaps most obvious) possibility is to break the approximate R-symmetry explicitly, but on general grounds [22] this generically leads to proximal SUSY vacua and some tuning has to be invoked (see e.g. [23-25] for models). One possibility that has been explored is to have explicit R-symmetry breaking only through Planck-suppressed couplings to messengers [26,27]. However, in this case the messenger scale and the messenger SUSY splittings come from different sources, so there are in general extra CP phases in the gaugino masses when doublet/triplet splitting of the messengers is taken into account. Similarly, messenger-parity is not guaranteed and there is a danger of inducing spontaneous breaking of the SM gauge group [28].

In this work we present another mechanism of spontaneous R-symmetry breaking which can be viewed as an improvement upon previous attempts, inasmuch as it is simple, more natural, dynamical and phenomenologically viable. The crucial observation is that all the abovementioned attempts at spontaneous R-symmetry breaking assumed that the R-charged pseudo-moduli obtain their potential at one-loop. In some instances, however, pseudo-moduli do not obtain their potentials until two-loops, and then the story can be quite different. Here, we will study one of the simplest illustrations of this phenomenon: SQCD with both massive and massless flavors. In this model, the meson composed of the massless quarks remains massless at one-loop in the low-energy effective theory [29]. Much above the strong coupling scale the theory exhibits nonperturbative runaway behavior; below the strong coupling scale, the two-loop potential was recently computed in the low-energy effective theory and was shown to be a monotonically decreasing function of the meson VEV [30]. Thus the model probably has runaway for all values of the field and has no vacuum at all.

In this paper we show how the runaway is automatically stabilized by “Planck” suppressed higher-dimension operators in the superpotential. The result is a local vacuum far from the origin but well below the cutoff scale of the effective low-energy theory, where both SUSY and R-symmetry are spontaneously broken.²

² A similar idea in a different context was proposed in [31] to explain the hierarchy between the Planck scale and the GUT scale. See also [29,32,33] for recent models based on massive SQCD where dangerous pseudo-moduli directions were stabilized with higher-dimension operators. However in these latter examples the pseudo-moduli in question were stabilized at the origin and were not the sources of R-symmetry breaking.

Aside from the SUSY-breaking minimum, the higher-dimension operators also restore a supersymmetric solution. We show that the distance between the supersymmetric and SUSY-breaking solution is parametrically large, ensuring that the metastable vacuum is parametrically long-lived.

Consequently, SQCD with massive and massless flavors has all the necessary ingredients, including large spontaneous R-symmetry breaking, for SUSY model building. We illustrate this by writing down the simplest application of our model, namely a complete model of “minimal” gauge mediation [8,34]. The role of the singlet coupling to messengers in our model is played by the meson composed of the massless electric quarks. As in [35], we couple the meson to the messengers through non-renormalizable interactions in the UV. We find that if the runaway is stabilized by a quartic coupling in the UV, surprisingly stringent constraints on such models of minimal gauge mediation come from the requirements of calculability in the SUSY-breaking sector together with non-tachyonic messengers.³ As we show further this model can be successfully retrofitted [36] and all the couplings can be explained in the context of discrete gauge symmetries [37].

Of course, the real promise of our new mechanism of dynamical SUSY and R-symmetry breaking is in its potential applications to models of direct gauge mediation, where there is no invariant distinction between messenger and SUSY-breaking fields. Since SQCD with massive and massless flavors is vector-like, it automatically comes equipped with large global flavor symmetries. It is easy to imagine gauging a subgroup of these flavor symmetries and identifying it with the SM gauge group. Then SUSY breaking will be automatically communicated to the SM via gauge mediation, and unlike in [6], the MSSM gauginos will acquire viable soft masses. In this context one still has to overcome the long-standing challenge of the Landau pole problem, since there will generally be lots of extra matter charged under the SM gauge group. This is a very interesting problem that we will reserve for future work.

Our paper is organized as follows. In section 2 we review SQCD with massive and massless flavors. We pay special attention to the theory far out along the pseudo-moduli space (where the runaway will ultimately be stabilized). In this regime, the theory can be described by an effective supersymmetric Lagrangian, and the leading-log approximation to

³ These constraints can be relaxed if one chooses to stabilize the runaway by even higher-dimension operators. The main drawback of this possibility is that a more complicated discrete symmetry is needed to maintain such a structure.

the runaway potential has been calculated as part of a more general analysis in [38], using the techniques of wavefunction renormalization [39,40]. However, to keep the discussion in this paper self-contained we present a sketch of the calculation in the appendix. In section 3 we introduce the stabilization of the two-loop runaway potential via higher-dimension operators, discuss the spectrum of particles around the SUSY-breaking vacuum, and derive constraints on the parameters of the model which follow from calculability and longevity. Section 4 is dedicated to constructing and analyzing a model of minimal gauge mediation using massive+massless SQCD as the SUSY-breaking sector.

2. SQCD with Massive and Massless Flavors

2.1. An overview of the model

In this section we review the main results of [30], while simultaneously establishing our notation and conventions (which differ slightly from those of [30]). We will consider SUSY QCD with $SU(N_c)$ gauge group and N_f flavors in the free magnetic phase ($N_c < N_f < 3N_c/2$). We will take $N_{f0} < N_c$ of the flavors to be massless with the other $N_{f1} = N_f - N_{f0}$ having equal mass.⁴ When all the flavors are massive (but light compared to the strong coupling scale Λ), this model exhibits a metastable supersymmetry breaking state [6]. However, when some of the flavors are massless, the situation is very different: some of the pseudo-moduli of the low-energy effective theory are not stabilized at one-loop [29]. Instead, they acquire a potential at two-loops and it was recently shown in [30] that this potential is monotonically decreasing along the pseudo-moduli space, leading to runaway behavior.

Now let us describe the low-energy effective theory in more detail. Using Seiberg duality [41], we have a weakly coupled description at low energies in terms of an IR-free $SU(N)$ gauge theory (with $N = N_f - N_c$). The matter content consists of a gauge singlet $N_f \times N_f$ meson matrix Φ and N_f flavors of magnetic quarks $\varphi, \tilde{\varphi}$. It will be convenient in the following to split the N_f flavor indices into N_{f0} and N_{f1} sized blocks, i.e.

$$\Phi = \begin{pmatrix} \Phi_{11} & \Phi_{10} \\ \Phi_{01} & \Phi_{00} \end{pmatrix}, \quad \varphi = \begin{pmatrix} \varphi_1 \\ \varphi_0 \end{pmatrix}, \quad \tilde{\varphi}^T = \begin{pmatrix} \tilde{\varphi}_1 \\ \tilde{\varphi}_0 \end{pmatrix}, \quad (2.1)$$

⁴ It is trivial to ensure the naturalness of this structure by imposing appropriate (discrete) symmetries which allow mass terms for some of the quarks but not others. We will have more to say about this in the next sections.

with Φ_{ij} an $N_{fi} \times N_{fj}$ matrix and φ_i , $\tilde{\varphi}_i^T$ $N_{fi} \times N$ matrices. The fields can be normalized to have canonical Kähler potential plus uncalculable higher-dimensional corrections suppressed by powers of Λ . In this normalization, the superpotential of the theory is

$$W = h \text{Tr} \Phi_{ij} \varphi_j \tilde{\varphi}_i - h \mu^2 \text{Tr} \Phi_{11} , \quad (2.2)$$

where h is an $\mathcal{O}(1)$ coupling and the traces are taken over the uncontracted flavor indices of the different fields. In what follows we will assume without loss of generality that all the couplings are real and positive. Note that although the global symmetry is no longer $SU(N_f)$, we are still denoting all the Yukawa couplings by the same symbol h – at the scale Λ this is the case up to corrections which vanish as $\frac{\mu}{\Lambda} \rightarrow 0$. Of course, below the scale Λ these couplings will run differently, and we will take this into account when necessary.

The representations of the fields under all the gauge and global symmetries preserved by the superpotential (2.2) are listed in the table below. Note that there are four independent $U(1)$ symmetries (including the R-symmetry), neglecting anomalies as in [6].

$SU(N)$	$SU(N_{f0})_L$	$SU(N_{f0})_R$	$SU(N_{f1})$	$U(1)_B$	$U(1)_1$	$U(1)_2$	$U(1)_R$	
Φ_{11}	1	1	1	adj \oplus 1	0	0	0	2
Φ_{10}	1	\square	1	$\bar{\square}$	0	-1	1	$3/2$
Φ_{01}	1	1	$\bar{\square}$	\square	0	1	0	$3/2$
Φ_{00}	1	\square	$\bar{\square}$	1	0	0	1	1
φ_1	\square	1	1	$\bar{\square}$	1	0	0	0
φ_0	\square	$\bar{\square}$	1	1	1	1	-1	$1/2$
$\tilde{\varphi}_1$	$\bar{\square}$	1	1	\square	-1	0	0	0
$\tilde{\varphi}_0$	$\bar{\square}$	1	\square	1	-1	-1	0	$1/2$

The reason for the funny $U(1)$ charges will become apparent in the next section.

Since $N_{f1} = N_f - N_{f0} > N_f - N_c$, rank conditions mean that SUSY is spontaneously broken at tree-level, as in [6]. The tree-level scalar potential is minimized along the following pseudo-moduli space:

$$\begin{aligned} \Phi_{11} &= \begin{pmatrix} 0 & 0 \\ 0 & X \end{pmatrix} , & \Phi_{10} &= \begin{pmatrix} 0 \\ Y \end{pmatrix} , & \varphi_1 &= \begin{pmatrix} \chi \\ 0 \end{pmatrix} , & \tilde{\varphi}_1^T &= \begin{pmatrix} \tilde{\chi} \\ 0 \end{pmatrix} , & \chi \tilde{\chi} &= \mu^2 \mathbb{1}_N , \\ \Phi_{01} &= (0 \quad \tilde{Y}) , & \text{arbitrary } \Phi_{00} , & \varphi_0 &= \tilde{\varphi}_0 &= 0 , \end{aligned} \quad (2.4)$$

with vacuum energy

$$V_0 = (N_{f1} - N)h^2\mu^4. \quad (2.5)$$

Here χ , $\tilde{\chi}$ are $N \times N$ matrices, X is an $(N_{f1} - N) \times (N_{f1} - N)$ matrix, Y and \tilde{Y}^T are $(N_{f1} - N) \times N_{f0}$ matrices, and Φ_{00} is an $N_{f0} \times N_{f0}$ matrix. At the origin of the pseudo-moduli space (2.4), the entire gauge symmetry and part of the global symmetry are broken:

$$SU(N) \times \left[SU(N_{f1}) \times U(1)_B \right] \rightarrow \left[SU(N)_D \times SU(N_{f1} - N) \times U(1)'_B \right], \quad (2.6)$$

with the other global symmetries remaining unaffected.

As mentioned at the beginning of this section, the main novelty introduced by the massless quarks is the fact that at one-loop, only some of pseudo-moduli are lifted by the Coleman-Weinberg potential [29]. Specifically, all the fields except Φ_{00} are lifted and stabilized at the origin by the one-loop potential. So to understand the dynamics of this model we are driven to a two-loop computation on the pseudo-moduli space parameterized by Φ_{00} . This has been considered in detail in [30]. There it was found that the origin of field space is *destabilized*, in other words, Φ_{00} acquires a negative mass squared term around the origin

$$V_{eff} = -h^2 \mu^2 \left(\frac{\alpha_h}{4\pi} \right)^2 N(N_c - N_{f0}) \left(1 + \frac{\pi^2}{6} - \log 4 \right) \text{Tr}(\Phi_{00}^\dagger \Phi_{00}) + \mathcal{O}(\Phi_{00}^4), \quad (2.7)$$

where $\alpha_h \equiv \frac{h^2}{4\pi}$. Moreover, it was shown in [30] that the two-loop potential for Φ_{00} is monotonically decreasing, leading to runaway behavior. This result has important physical implications for ISS based model building and it can be interpreted geometrically in the appropriate brane configuration [30]. In the next subsection we will study the model in more detail in the $\langle \Phi_{00} \rangle \gg \mu$ regime.

2.2. Effective theory at large $\langle \Phi_{00} \rangle$

Since the pseudo-modulus Φ_{00} has a runaway potential, we are motivated to study the theory in the large $\langle \Phi_{00} \rangle \gg \mu$ regime. There the analysis of the pseudo-modulus potential simplifies considerably – we can use the wavefunction renormalization techniques of [39,40] to compute the two-loop potential in the leading-log approximation using only one-loop anomalous dimensions.

To be more precise, we are interested in the theory for generic $\langle \Phi_{00} \rangle$ in the regime

$$h\mu \ll h\langle \Phi_{00} \rangle \ll \Lambda. \quad (2.8)$$

The point is that below the scale $h\langle\Phi_{00}\rangle$ we can integrate out supersymmetrically the N_{f0} magnetic quarks coupling to Φ_{00} . Then we are left with an effective supersymmetric theory of the form

$$\begin{aligned} K_{eff} &= \mathcal{Z}_{\Phi_{ij}} \text{Tr} \Phi_{ij}^\dagger \Phi_{ij} + \mathcal{Z}_{\varphi_1} (\text{Tr} \varphi_1^\dagger \varphi_1 + \text{Tr} \tilde{\varphi}_1^\dagger \tilde{\varphi}_1) + \dots, \\ W_{eff} &= h \text{Tr} \Phi_{11} \varphi_1 \tilde{\varphi}_1 - h \mu^2 \text{Tr} \Phi_{11} - h \text{Tr} \tilde{\varphi}_1 \Phi_{10} \Phi_{00}^{-1} \Phi_{01} \varphi_1, \end{aligned} \quad (2.9)$$

where the wavefunction factors in the Kähler potential are functions of Φ_{00} and where \dots includes both the uncalculable $\mathcal{O}(1/\Lambda)$ corrections from the duality together with the calculable $\mathcal{O}(1/\langle\Phi_{00}\rangle)$ corrections from integrating out $\varphi_0, \tilde{\varphi}_0$.

The effective theory still has an $SU(N)$ gauge symmetry; the gauge coupling can be either IR or asymptotically free depending on whether $N_{f1} > 3N$ or $N_{f1} < 3N$, respectively. The latter would be problematic because then the Kähler potential would no longer be well-behaved around the origin where we need to compute loops to understand the SUSY-breaking vacuum. Thus in order to have a reliable SUSY-breaking vacuum we must require

$$N_{f1} > 3N = 3(N_f - N_c). \quad (2.10)$$

This goes beyond the usual free-magnetic phase requirement $N_f > 3N$. Note that (2.10) is equivalent to

$$N_{f0} < 3N_c - 2N_f < N_c. \quad (2.11)$$

The effective theory is essentially the “macroscopic model” of [6] but with a wavefunction factor in the Kähler potential that depends on Φ_{00} . The theory still breaks SUSY via the rank condition and stabilizes all the pseudo-moduli. The vacuum energy is:

$$V_0 = (N_{f1} - N)h^2 \mu^4 \mathcal{Z}_{\Phi_{11}}^{-1}. \quad (2.12)$$

This can be interpreted as the effective potential for Φ_{00} , after using the RGEs to compute $\mathcal{Z}_{\Phi_{11}}$ in the leading-log approximation. The computation of $\mathcal{Z}_{\Phi_{11}}$ has been performed in [38] as a special case of a more general survey of pseudo-moduli. Here we simply quote the answer; we will reproduce the calculation in appendix A for the sake of completeness.

$$\log \mathcal{Z}_{\Phi_{11}} = N \left(\frac{\alpha_h}{4\pi} \right)^2 \text{Tr} \left(\log \frac{\Phi_{00}^\dagger \Phi_{00}}{\mu^2} \right)^2 + \dots, \quad (2.13)$$

where \dots are subleading in the two-loop leading-log approximation. Plugging this into (2.12), we have in the leading-log approximation

$$V_{eff} = -(N_{f1} - N)N h^2 \mu^4 \left(\frac{\alpha_h}{4\pi} \right)^2 \text{Tr} \left(\log \frac{\Phi_{00}^\dagger \Phi_{00}}{\mu^2} \right)^2. \quad (2.14)$$

Of course, (2.14) can also be computed more directly by taking the large field limit of the two-loop potential computed in [30].

3. Stabilizing the Runaway

3.1. Adding a small quartic term

In this section, we will show how the runaway can be turned into a metastable SUSY-breaking vacuum by allowing for generic non-renormalizable superpotential interactions. One can easily imagine that such terms are present at the Planck scale, generated by the theory of quantum gravity. Alternatively, they could be trivially generated at an intermediate scale by integrating out massive matter in some renormalizable UV completion. In any event, the point is that, since the runaway direction is a dimension two meson in the electric theory, the lowest-dimension non-renormalizable operators – dimension four interactions amongst the electric quarks – become mass terms for the runaway fields in the IR. So the runaway is automatically stabilized by generic higher-dimensional superpotential interactions.

Instead of considering the most general quartic terms for the electric quarks, we will focus only those involving the N_{f0} massless quarks; these become in the IR:

$$\delta W = \frac{1}{2} m_{f_1 g_1 f_2 g_2} (\Phi_{00})_{f_1 g_1} (\Phi_{00})_{f_2 g_2} . \quad (3.1)$$

It would be interesting to study in detail more general deformations involving Φ_{01} , Φ_{10} and Φ_{11} , but we will not do so in this paper. We will briefly discuss the possible effects of such deformations at the end of the next subsection. The upshot is that, while such deformations may complicate the details of our analysis, it is not obvious that they will change the qualitative conclusions.

For $N_{f0} > 2$, the quartic deformations in (3.1) necessarily break the $SU(N_{f0})_L \times SU(N_{f0})_R$ symmetry down to a subgroup. For simplicity, we will choose to preserve $SU(N_{f0})_D$. We will further simplify the analysis by considering only one of the two independent deformations that respect this symmetry:

$$\delta W = \frac{1}{2} h \epsilon \mu \text{Tr} \Phi_{00}^2 . \quad (3.2)$$

One could of course consider an arbitrary mixture of (3.2) and the other $SU(N_{f0})_D$ symmetric deformation $(\text{Tr} \Phi_{00})^2$. However, since this complication will have no effect on the essential physics that we will discuss below, we will ignore it. Notice that in (3.2) we have written the deformation parameter in units of $h\mu$, with a new dimensionless parameter ϵ . This will simplify the notation throughout.

In order for the deformation to not overwhelm the negative two-loop mass-squared for Φ_{00} (2.7) around the origin, we need ϵ to be a small parameter (as its name suggests), specifically

$$\epsilon \lesssim \frac{\alpha_h}{4\pi} . \quad (3.3)$$

Ordinarily this small parameter would seem rather artificial; however, here the UV completion can make it natural – all we need is that the UV scale that suppresses (3.2) in the electric theory is sufficiently large compared to Λ .

We are interested in whether the deformation can stabilize the runaway at large $\Phi_{00} \gg \mu$. In this regime, the effective theory (2.9) deformed by (3.1) is a valid description. Thus the vacuum energy along the pseudo-moduli space Φ_{00} is

$$V_0 = (N_{f1} - N)h^2\mu^4\mathcal{Z}_{\Phi_{11}}^{-1} + h^2\epsilon^2\mu^2\text{Tr } \Phi_{00}^\dagger\Phi_{00} . \quad (3.4)$$

Here we are neglecting the wavefunction renormalization of Φ_{00} itself – since we are going to be balancing the two terms in (3.4) against each other to find the metastable vacuum, any corrections from $\mathcal{Z}_{\Phi_{00}}$ are obviously going to be subleading.

Substituting the calculation (2.14) of the previous section, it is straightforward to minimize the potential,

$$V_{eff} = -(N_{f1} - N)Nh^2\mu^4 \left(\frac{\alpha_h}{4\pi} \right)^2 \text{Tr} \left(\log \frac{\Phi_{00}^\dagger\Phi_{00}}{\mu^2} \right)^2 + h^2\epsilon^2\mu^2\text{Tr } \Phi_{00}^\dagger\Phi_{00} , \quad (3.5)$$

and find the local minimum at

$$\langle \Phi_{00} \rangle \approx \frac{\mu}{\delta} \sqrt{\log \frac{1}{\delta}} \mathbb{1}_{N_{f0}} , \quad \frac{1}{\delta} \equiv \sqrt{(N_{f1} - N)N} \left(\frac{\alpha_h}{2\pi} \right) \frac{1}{\epsilon} . \quad (3.6)$$

The corrections to (3.6) are of the form $\langle \Phi_{00} \rangle \rightarrow \langle \Phi_{00} \rangle \times \left(1 + \mathcal{O} \left(\frac{1}{\log \delta}, \frac{\log(\log \delta)}{\log \delta} \right) \right)$ and they come not only from minimizing the potential (3.5), but also from the subleading-log corrections to the wavefunction $\mathcal{Z}_{\Phi_{11}}$. It is very important (and fortunate) for our purposes that with the leading-log potential alone, we can capture the location of the Φ_{00} VEV (3.6) up to small corrections.

In the regime $\epsilon \ll \frac{\alpha_h}{4\pi}$, the minimum of the potential is indeed located at $\langle \Phi_{00} \rangle \gg \mu$, which ensures that the approximation used to obtain the minimum is self-consistent.⁵

⁵ Note that this is a stronger assumption than (3.3). In fact, (3.3) is sufficient for the existence of a minimum away from the origin – since the potential turns up at large fields, as long as the origin is destabilized such a minimum must exist. In this paper we will only consider the more restricted case $\epsilon \ll \frac{\alpha_h}{4\pi}$ where the effective theory is valid.

Another requirement comes from demanding $\Phi_{00} \ll \Lambda$, so that the effective description in terms of the magnetic theory can be trusted. According to (3.6), this implies

$$\epsilon \gg \frac{\alpha_h}{4\pi} \frac{\mu}{\Lambda} . \quad (3.7)$$

(In fact, we will see below that an even stronger bound is required for calculability.) Since these requirements (3.3), (3.7) can obviously be satisfied for the proper choice of parameters, we conclude that a viable minimum of the potential can be obtained by balancing the two-loop runaway potential against a tree-level mass term for Φ_{00} .

3.2. Properties of the metastable vacuum

Now let us describe the properties of the metastable vacuum in more detail. The deformation (3.2) explicitly breaks some of the global symmetries (2.3). The remaining symmetries are:

$$SU(N) \times \left[SU(N_{f0})_D \times SU(N_{f1}) \times U(1)_B \times U(1)_1 \times U(1)_R \right] . \quad (3.8)$$

Note that the choice of $U(1)$ charges in (2.3) makes the symmetry breaking here especially simple.

In the metastable vacuum, the symmetries are broken spontaneously by $\langle \Phi_{00} \rangle \neq 0$ and $\varphi_1, \tilde{\varphi}_1 \neq 0$ to

$$\begin{aligned} SU(N) \times & \left[SU(N_{f0})_D \times SU(N_{f1}) \times U(1)_B \times U(1)_1 \times U(1)_R \right] \\ & \rightarrow \left[SU(N)_D \times SU(N_{f0})_D \times SU(N_{f1} - N) \times U(1)'_B \times U(1)_1 \right] . \end{aligned} \quad (3.9)$$

Thus we see the crucial difference with [6]: the $U(1)_R$ symmetry is completely broken in the metastable vacuum! This can have nice applications to phenomenology, as we will see in the next section.

There is a rich spectrum of fluctuations around the metastable vacuum. We will now describe these, roughly in order from heaviest to lightest.

1. First of all, the quark superfields $\varphi_0, \tilde{\varphi}_0$ are very heavy with $\mathcal{O}(h\langle \Phi_{00} \rangle)$ masses. At the tree-level, the squarks have small SUSY splittings $\delta m^2 \sim h^2 \epsilon \mu \langle \Phi_{00} \rangle$.
2. From the description in terms of the effective theory (2.9), (3.4), we see that the spectrum below the scale $h\langle \Phi_{00} \rangle$ basically factorizes into an ISS-like spectrum for $\Phi_{11}, \varphi_1, \tilde{\varphi}_1$ plus the spectrum of the additional modes $\Phi_{01}, \Phi_{10}, \Phi_{00}$. The fields in

the ISS sector have masses $\sim h\mu$ (except for the pseudo-moduli, which have one-loop suppressed masses-squared, and the Goldstone bosons). Some of the fields have large $\delta m^2 \sim h^2\mu^2$ splittings. Since these fields have been given a detailed description in [6], we will move on to describing the new fields.

3. Let us split up Φ_{01} and Φ_{10} into two submatrices each:

$$\Phi_{01} = \begin{pmatrix} \tilde{A} & \tilde{Y} \end{pmatrix}, \quad \Phi_{10} = \begin{pmatrix} A \\ Y \end{pmatrix}, \quad (3.10)$$

where A, \tilde{A}^T are $N \times N_{f0}$ and Y, \tilde{Y}^T are $(N_{f1} - N) \times N_{f0}$. Then from the third term in the effective superpotential (2.9), we see that A, \tilde{A} acquire masses $\sim h\mu^2/\langle\Phi_{00}\rangle$ from the VEV of φ_1 . They also have SUSY splittings $\delta m^2 \sim \frac{h\mu^2}{\langle\Phi_{00}\rangle^2} F_{\Phi_{00}} \sim \frac{h^2\epsilon\mu^3}{\langle\Phi_{00}\rangle}$. In order for these to not make the scalar components of A, \tilde{A} tachyonic, $\langle\Phi_{00}\rangle$ cannot be too large: $\langle\Phi_{00}\rangle \ll \mu/\epsilon$. Fortunately, this is guaranteed by the formula for the metastable vacuum (3.6)!

4. Meanwhile Y, \tilde{Y} are massless at tree-level. Since they have a suppressed coupling $\sim h\mu/\langle\Phi_{00}\rangle$ to the SUSY-split “messenger” fields in $\varphi_1, \tilde{\varphi}_1$, their one-loop mass-squared will be similarly suppressed, $m^2 \sim (\frac{h\mu}{\langle\Phi_{00}\rangle})^2 (h\mu)^2$. An explicit calculation shows this mass-squared is positive. Regarding fermions in this sector, the mass term $\psi_Y \psi_{\tilde{Y}}$ is allowed by the unbroken symmetries and it is indeed generated at one-loop. As before, to generate this operator we have to go through propagators which are suppressed by $1/\langle\Phi_{00}\rangle$.
5. Finally, Φ_{00} itself obviously has a mass $\sim h\epsilon\mu$, except for $\text{Im}(\text{Tr } \Phi_{00})$ which is the massless Goldstone boson of $U(1)_R$ breaking.⁶

Finally, let us comment on the potential effects of other quartic operators besides the ones considered in (3.1). From the discussion of the spectrum, it becomes clear that adding operators of the type $\epsilon\mu\Phi_{11}\Phi_{10}, \epsilon\mu\Phi_{11}\Phi_{01}$ to the superpotential will change our vacuum significantly. The reason is that the F -term of Φ_{11} gives rise to a tadpole for the light Y

⁶ Actually, the effective scalar theory (3.4) appears to possess the full $SU(N_{f0})^2$ symmetry even though the superpotential (3.2) breaks it down to $SU(N_{f0})_D$. So when $\langle\Phi_{00}\rangle \neq 0$ it would seem there should be many more tree-level massless modes than just the Goldstone boson of $U(1)_R$ breaking. In fact, these accidental massless modes are all lifted by considering the most general $SU(N_{f0})_D$ -symmetric deformation, instead of just (3.2).

modes, and shifts them far away from the origin. These operators might not change the qualitative conclusions, but for our analysis to strictly apply they have to be absent for some reason, e.g. by a $SU(N_{f0})_D \times SU(N_{f1})$ global symmetry or discrete symmetries like those we consider in section 4. By the same reasoning, operators of the form $\epsilon\mu\Phi_{11}\Phi_{00}$ are also dangerous, so they should also be forbidden by symmetries. In fact, these operators are automatically forbidden by any symmetry which allows a linear term in Φ_{11} , while simultaneously forbidding Φ_{00} and allowing Φ_{00}^2 .

3.3. Calculability, nonperturbative effects and lifetime

Finally, let us briefly address various issues related to the consistency of our analysis of the metastable vacuum: calculability, irrelevancy of the nonperturbative effects, and the lifetime.

First, because the tree-level mass of Φ_{00} is so much smaller than μ , we need to be careful that uncalculable corrections to the Kähler potential do not overwhelm the tree-level superpotential for Φ_{00} . In particular, terms like

$$K_{eff} \supset \frac{c}{\Lambda^2} \Phi_{11}^\dagger \Phi_{11} \Phi_{00}^\dagger \Phi_{00} \quad (3.11)$$

in the Kähler potential, with c an uncalculable $\mathcal{O}(1)$ number, contribute to the mass of Φ_{00} via $F_{\Phi_{11}} \sim h\mu^2$:

$$V_{eff} \supset \frac{c}{\Lambda^2} (h\mu^2)^2 \Phi_{00}^\dagger \Phi_{00} . \quad (3.12)$$

In order for this to be negligible compared with the tree-level mass $(h\epsilon\mu)^2$, we clearly need

$$\epsilon \gg \frac{\mu}{\Lambda} . \quad (3.13)$$

Note that this is a *stronger* requirement than (3.7) which we deduced above by demanding that $\langle \Phi_{00} \rangle \ll \Lambda$. We will get back to this constraint in more detail in section 4.

Next, the issue of whether nonperturbative effects can be neglected around the metastable vacuum boils down to the IR-freedom requirement (2.10), just as in [6]. In more detail, when Φ_{00} and Φ_{11} have generic VEVs, then by the standard scale matching argument the dynamical superpotential is

$$W_{dyn} = N \left(\frac{h^{N_f} \det \Phi_{00} \det \Phi_{11}}{\Lambda^{N_f - 3N}} \right)^{1/N} . \quad (3.14)$$

The calculability requirement (2.10) also guarantees that the non-perturbative superpotential is an irrelevant ($\dim > 3$) operator around $\Phi_{11} = 0$ even for $\Phi_{00} \neq 0$. Thus it can be safely ignored when computing loops around the non-SUSY vacuum.

Finally, let us address the issue of the lifetime. For this we need to know where the SUSY vacua are. At this point we will take $h = 1$ to simplify the expressions, because the value of h does not affect our analysis of the lifetime. Taking the ansatz $\Phi_{00} \propto \mathbb{1}_{N_{f0}}$ and $\Phi_{11} \propto \mathbb{1}_{N_{f1}}$ (and all other fields zero), it is straightforward to set the F -terms of the magnetic superpotential to zero (including (3.14)) and solve. Introducing the additional small parameter

$$\tilde{\epsilon} = \frac{\mu}{\epsilon \Lambda} , \quad (3.15)$$

this yields (ignoring phases)

$$\begin{aligned} \langle \Phi_{00} \rangle_{SUSY} &= \tilde{\epsilon}^{-\frac{3N_c - 2N_f}{2N_c - N_{f0}}} \epsilon^{-\frac{3N_c - 2N_f + N_c - N_{f0}}{2N_c - N_{f0}}} \mu = \tilde{\epsilon}^{\frac{N_{f1} + N}{2N_c - N_{f0}}} \epsilon^{\frac{2N}{2N_c - N_{f0}}} \Lambda , \\ \langle \Phi_{11} \rangle_{SUSY} &= \tilde{\epsilon}^{-\frac{2(3N_c - 2N_f)}{2N_c - N_{f0}}} \epsilon^{-\frac{2(3N_c - 2N_f) - N_{f0}}{2N_c - N_{f0}}} \mu . \end{aligned} \quad (3.16)$$

So we always have

$$\mu \ll \langle \Phi_{00} \rangle_{SUSY} \ll \Lambda , \quad \langle \Phi_{11} \rangle_{SUSY} \gg \mu . \quad (3.17)$$

Since $\langle \Phi_{11} \rangle_{non-SUSY} = 0$, (3.17) implies that the SUSY vacuum is always separated by an amount $\Delta\Phi_{11} \gg \mu$ from the non-SUSY vacuum. This is enough to ensure a parametrically long-lived metastable state, provided that the barrier also scales with $\Delta\Phi_{11}$. The following argument shows that this must be the case. As in [6], the most efficient path between the non-SUSY and SUSY vacua is to first climb up to a local extremum with $q = \tilde{q} = 0$ and $V = N_{f1}\mu^4$. Then the potential is schematically (now setting $\Lambda = 1$)

$$V \sim N_{f1} \left| \Phi_{00}^{N_{f0}/N} \Phi_{11}^{N_{f1}/N-1} - \mu^2 \right|^2 + N_{f0} \left| \Phi_{00}^{N_{f0}/N-1} \Phi_{11}^{N_{f1}/N} - \epsilon \mu \Phi_{00} \right|^2 . \quad (3.18)$$

Now the question is how far does one have to go in field space before the potential decreases to $V_{non-SUSY} = (N_{f1} - N)\mu^4$? Since the vacuum energy comes primarily from the first term in (3.18), at the very least we have to decrease it by making

$$\Phi_{00}^{N_{f0}/N} \Phi_{11}^{N_{f1}/N-1} \sim \mu^2 . \quad (3.19)$$

Consider now all values of Φ_{00} and Φ_{11} which satisfy (3.19). We claim that of all such values, either $\Phi_{00} \gg \mu$ or $\Phi_{11} \gg \mu$. Indeed, if $\Phi_{11} \lesssim \mu$, then $\Phi_{00} \gtrsim \mu^{(3N - N_{f1})/N_{f0}}$ and one can check that this satisfies $\Phi_{00} \gg \langle \Phi_{00} \rangle_{non-SUSY}$, $\langle \Phi_{00} \rangle_{SUSY} \gg \mu$. Therefore the width of the potential barrier is guaranteed to be parametrically larger than μ , either through $\Delta\Phi_{11} \gg \mu$ or through $\Delta\Phi_{00} \gg \mu$. This in turn ensures a parametrically long lifetime for the metastable vacuum.

3.4. Summary of the requirements on the metastable vacuum

Since the discussion has been technical at times, we would like to conclude this section by collecting all the different requirements that need to be satisfied in order for the metastable vacuum to exist.

In addition to requiring that the theory is in the free magnetic phase ($N_c < N_f < \frac{3}{2}N_c$), we also must require

$$N_{f0} < 3N_c - 2N_f \quad \iff \quad N_{f1} > 3(N_f - N_c) \quad (3.20)$$

in order for the non-perturbative effects to be negligible around the metastable vacuum, and to ensure a parametrically long lifetime.

The parameter $\epsilon = m_{\Phi_{00}}/h\mu$ must satisfy the constraints:

$$\frac{\mu}{\Lambda} \ll \epsilon \lesssim \frac{\alpha_h}{4\pi} . \quad (3.21)$$

The lower bound comes from demanding that uncalculable Kähler potential corrections can be neglected relative to the tree-level mass term. It also ensures that $\langle \Phi_{00} \rangle \ll \Lambda$. The upper bound comes from requiring that the tree-level mass term does not overwhelm the two-loop potential and stabilize the metastable vacuum at the origin. We will be assuming a stronger bound $\epsilon \ll \frac{\alpha_h}{4\pi}$ throughout the paper so that the leading-log approximation to the effective potential (3.5) can be used.

4. A Simple Model Building Application

4.1. The model

While the main emphasis of our paper is to present a new mechanism of metastable supersymmetry-plus-R-symmetry breaking, in this section we would also like to briefly touch upon how this mechanism can be applied to building viable and natural models of gauge mediation. (A more detailed study of the model building applications, in particular to models of direct gauge mediation, will be reserved for a future publication.) To that end, we will present here the simplest application of our mechanism to model building: using our SUSY-breaking sector in a model of “ordinary” or “minimal” gauge mediation [8,34]. Although this may seem like a trivial application, we will see that various consistency conditions and phenomenological requirements force the model to be surprisingly predictive.

Our SUSY-breaking sector consists of the theory we studied in great detail in the previous sections. For simplicity, we assume the minimum number of massless electric quarks, $N_{f0} = 1$. We will also assume for simplicity that $h = 1$; in general, it is not a free parameter but is instead determined by various (uncalculable) $\mathcal{O}(1)$ numbers coming from Seiberg duality [6].

The coupling to the messengers is as in ordinary gauge mediation – we couple the field Φ_{00} (which has both a lowest-component and an F -component VEV) to a vector-like pair of messenger fields $\Psi, \tilde{\Psi}$ transforming in $5, \bar{5}$ of $SU(5)$ GUT:

$$W_{mess} = \lambda \Phi_{00} \Psi \tilde{\Psi} . \quad (4.1)$$

Here λ is some dimensionless coupling; since it is actually a non-renormalizable interaction in the UV, λ will be naturally quite small. We are considering only one messenger pair here; the generalization to arbitrary numbers of messengers is trivial and does not affect the rough sketch of the physics presented here.

The overall scale of the model is set by phenomenological considerations. We wish to obtain a realistic spectrum of SUSY partners, so we set the gaugino mass to be around 100 GeV. This is achieved by using the usual formula of ordinary gauge-mediation

$$\frac{\alpha_r}{4\pi} \frac{F_{\Phi_{00}}}{\Phi_{00}} \sim 10^2 \text{ GeV} \implies \epsilon \mu \sim 10^5 \text{ GeV} . \quad (4.2)$$

Now let us consider the various consistency conditions on the model. In addition to the constraints (3.20), (3.21) on the SUSY breaking sector, we also have the following constraint coming from the requirement of non-tachyonic messengers:

$$(\lambda \Phi_{00})^2 > \lambda F_{\Phi_{00}} . \quad (4.3)$$

Substituting $F_{\Phi_{00}} = \epsilon \mu \Phi_{00}$, this becomes

$$\lambda \Phi_{00} > \epsilon \mu . \quad (4.4)$$

If λ were an $\mathcal{O}(1)$ coupling then this constraint would be trivial to satisfy, since $\Phi_{00} \gg \mu$. However, since λ comes from a non-renormalizable interaction in the UV, we find that $\lambda \ll 1$ in such a way that (4.4) becomes quite restrictive, similar to [35].

To see this in more detail, let the Φ_{00}^2 operator be suppressed by a mass scale M_1 in the electric theory, and the $\Phi_{00} \Psi \tilde{\Psi}$ be suppressed by a (possibly different) mass scale M_2 . Then we have

$$\lambda \sim \frac{\Lambda}{M_2} , \quad \epsilon \sim \frac{\Lambda^2}{M_1 \mu} , \quad (4.5)$$

and (4.4) is equivalent to (after substituting $\Phi_{00} \sim \frac{C\mu}{\epsilon} \frac{\alpha_h}{4\pi}$ with C the combination of color and flavor factors and the log appearing in (3.6))

$$\frac{M_2}{CM_1} < \frac{\mu}{\Lambda\epsilon} \frac{\alpha_h}{4\pi} \ll 1 . \quad (4.6)$$

Thus for $C \sim \mathcal{O}(1-10)$ there must be a sizeable hierarchy between M_2 and M_1 . In practice this turns out to be at least a factor of $M_2/M_1 \sim 10^{-4}$. This fine-tuning in M_2 vs. M_1 is perhaps the least attractive feature of this model.⁷

We can further manipulate the constraints to derive the allowed ranges of the parameters. We start with (4.2) and (4.5), which together imply

$$\epsilon\mu \sim \frac{\Lambda^2}{M_1} \sim 10^5 \text{ GeV} . \quad (4.7)$$

Translating (3.21) to the new parameters leads to

$$\frac{\mu}{\Lambda} \ll \frac{\Lambda^2}{M_1\mu} \lesssim \frac{1}{(4\pi)^2} , \quad (4.8)$$

where we have taken $h \sim 1$ for the upper inequality. Combining this with (4.7) we obtain

$$\begin{aligned} \mu &\gtrsim 10^7 \text{ GeV} , \\ \Lambda &\gg \frac{\mu^2}{10^5 \text{ GeV}} \gtrsim 10^9 \text{ GeV} , \\ M_1 &\gg \frac{\mu^4}{10^{15} \text{ GeV}^3} \gtrsim 10^{13} \text{ GeV} . \end{aligned} \quad (4.9)$$

In fact, if we do not want M_1 to exceed the Planck scale, then according to (4.9), μ is actually bounded above by $\sim 10^8$ GeV. So we see the parameters can only take values in the ranges

$$\mu \sim 10^7-10^8 \text{ GeV} , \quad \Lambda \sim 10^9-10^{12} \text{ GeV} , \quad M_1 \sim 10^{13}-10^{19} \text{ GeV} . \quad (4.10)$$

⁷ It is interesting to note that, if instead of a quartic deformation, the leading non-renormalizable term is an even higher-dimension operator, $\delta W \sim \frac{\Lambda^k}{M_1^{2k-3}} \Phi_{00}^k$, (and assuming the messengers are still coupled through (4.1)) the constraint of non-tachyonic messengers becomes trivial for $k \geq 3$. For instance, if $k = 3$ the messengers are non-tachyonic as long as

$$\frac{M_1}{M_2} > \frac{\Lambda^2}{M_1^2} .$$

This is a rather trivial inequality which does not lead to any hierarchy between M_1 and M_2 .

Apparently, the parameters of the model are quite constrained by all the consistency conditions, and the result is a relatively predictive version of ordinary gauge mediation!

Let us conclude this subsection by just mentioning one particularly intriguing choice of parameters that satisfies all the constraints (and in fact minimizes the hierarchy between M_2 and M_1):

$$\mu \sim 10^8 \text{ GeV}, \quad \Lambda \sim 10^{12} \text{ GeV}, \quad M_1 \sim M_{\text{pl}} \sim 10^{19} \text{ GeV}. \quad (4.11)$$

This satisfies all the requirements as long as $M_2 \lesssim 10^{15}$ GeV. Interestingly, this is tantalizingly close to the GUT scale, so perhaps one could imagine a UV completion where GUT-scale fields connect Φ_{00} to the messengers, but only Planck-suppressed interactions generate Φ_{00}^2 .

4.2. Phenomenological features of the model

Now let us briefly mention some phenomenological aspects of our ordinary gauge mediation model. For concreteness we will focus on the point (4.11) with $M_2 \sim 10^{15}$ GeV. However, we would like to emphasize that the statements we will make in this subsection are for the most part *predictions* of our model, not simply consequences of the particular choice of parameters.

In our model, the parameters of ordinary gauge mediation are

$$\begin{aligned} M_{\text{mess}} &\sim \lambda \Phi_{00} \sim 10^5 - 10^6 \text{ GeV}, \\ F &\sim \lambda F_{\Phi_{00}} \sim 10^{10} - 10^{11} \text{ GeV}^2, \\ m_{3/2} &\sim \frac{F_{\Phi_{11}}}{M_{\text{pl}}} \sim 1 - 10 \text{ MeV}. \end{aligned} \quad (4.12)$$

Here we have used $\lambda \sim 10^{-3}$ and $\epsilon \sim 10^{-3}$. Note that M_{mess} and F come from the same source, the R-symmetry breaking scale Φ_{00} , and therefore there are no extra CP phases in the gaugino masses even if doublet/triplet splitting of the messengers is present. Moreover, having one and the same source for M_{mess} and F , as we do, guarantees messenger-parity, which is necessary for a consistent spectrum of soft terms.

Since R-symmetry is spontaneously broken, the R-axion is massless in gauge theory, up to small non-perturbative contributions. On top of the field theory effects, there is also a supergravity contribution to the mass m_a . Following [42] we find that this contribution is

$$m_a \sim \sqrt{\frac{\epsilon \mu^3}{M_{\text{pl}}}} \sim 10 \text{ GeV}. \quad (4.13)$$

This is already well above the astrophysical bound, $m_a \gtrsim 10$ MeV.

Note that the mass of the gravitino in (4.12) is always determined by the largest F -term in the SUSY-breaking sector. In this model, this is not $F_{\Phi_{00}}$ but is instead $F_{\Phi_{11}} \sim \mu^2 \sim 10^{16}$ GeV². So what we have is a model with light messengers but a heavy (~ 1 MeV) gravitino. This combination of features is generally rare in models of gauge-mediation; here it is achieved because the hidden sector has a subsector with much larger SUSY-breaking than the SUSY-breaking felt by the messengers.⁸

A heavy gravitino can have observable consequences at colliders. In general, a gravitino mass of (4.12) implies that the NLSP is sufficiently long lived that it will escape the detector. (For a good review of this subject and more references, we refer the reader to [2].) Thus the standard $\gamma\gamma$ +MET signal of gauge mediation will not be observed. On the other hand, if the NLSP is a very long-lived slepton (as would be the case, for instance, if the messenger scale is very low or there are $N_m \sim 5$ messengers) then the collider signatures of the model could be quite spectacular.

4.3. Naturalizing the UV model

To build a natural model (starting from some high scale) with the dynamics described above we clearly need to forbid all the super-renormalizable terms in the UV Lagrangian, while still allowing for the quartic interactions between the massless electric quarks. In addition, as explained in section 3, some quadratic operators must be forbidden for our analysis to be reliable. We would also like to forbid the most general couplings between the electric quarks and the messengers while allowing for (4.1); otherwise the messenger SUSY masses and F -terms will no longer necessarily come from the same source. Finally, we would like to generate the mass scale μ dynamically. As we will see in this section, all four of these conditions can be met by “retrofitting the model” [36] with an auxiliary gauge group and imposing its associated non-anomalous discrete R-symmetry. This approach was implemented recently in a similar MGM-type context by [27].

The main idea of retrofitting is as follows [36]. Consider a pure super Yang-Mills (SYM) theory $SU(\tilde{N})$ with the strong scale being $\tilde{\Lambda}$. At energies below $\tilde{\Lambda}$ there is a

⁸ The gravitino mass also sets the scale of all the gravity mediated contributions to soft terms. These contributions are not degenerate and, in general, violate flavor in the maximal possible way. For the gravitino masses considered here, there is no contradiction with precision measurements and these contributions are allowed to be arbitrary. For a recent work analyzing flavor aspects of combined gauge and gravity mediation see [43].

gaugino condensate which can be used to source masses for some of the electric quarks. Furthermore, the SYM theory has a non-anomalous $\mathbb{Z}_{2\tilde{N}}$ R-symmetry, and this can be promoted to a gauged discrete symmetry of the entire theory.

Now let us turn to a concrete realization of the UV theory along these lines. Consider the following gauge symmetry:

$$SU(3) \times SU(N_c) \times G_{SM} \times \mathbb{Z}_6 . \quad (4.14)$$

The $SU(3)$ group is a pure SYM theory, G_{SM} is the SM (or GUT) gauge group and $SU(N_c)$ is the gauge group of SQCD. Let Q_1 denote the N_{f1} massive electric quarks of SQCD and Q_0 be the only electric quark which is massless in the UV. \mathbb{Z}_6 is generated by g such that $g^6 = 1$. We assign the following charges under \mathbb{Z}_6 :

$$\int d^2\theta : \quad g^{-2} ; \quad W_\alpha W^\alpha, \quad Q_0, \quad \tilde{Q}_0, \quad \Psi, \quad \tilde{\Psi} : \quad g^2 ; \quad Q_1, \quad \tilde{Q}_1 : \quad g^3 . \quad (4.15)$$

Here W_α is the chiral field strength of the $SU(3)$ SYM. Note that these charge assignments forbid all the renormalizable couplings. Now let us write down the first few terms in the superpotential of this theory:

$$W = \frac{1}{M_1} (Q_0 \tilde{Q}_0)^2 + \frac{1}{M_2} Q_0 \tilde{Q}_0 \Psi \tilde{\Psi} + \frac{1}{M_3^2} \text{Tr} W_\alpha W^\alpha Q_1 \tilde{Q}_1 + \dots . \quad (4.16)$$

Here \dots stand for operators of dimension 6 and higher, as well as for quartic messenger self-interactions, both of which are irrelevant for our discussion. We see that the dangerous terms $Q_1 \tilde{Q}_1 \Psi \tilde{\Psi}$ are forbidden by the discrete R-symmetry. We also see that all but the Φ_{00}^2 -type quartic interactions are forbidden.⁹

To estimate $\tilde{\Lambda}$ we take for simplicity $M_3 \sim M_{\text{Pl}}$. For $\mu^2 \sim 10^{16}$ GeV² (4.11), we find $\tilde{\Lambda} \sim 10^{14}$ GeV. Hence $\tilde{\Lambda} \gg \Lambda$, which is necessary for the consistency of our analysis. Gaugino condensation breaks \mathbb{Z}_6 to \mathbb{Z}_2 , which is an unbroken symmetry of the theory. Indeed, $\Phi_{00} \sim Q_0 \tilde{Q}_0$, which obtains a VEV, is R-even. Note that this \mathbb{Z}_2 R-symmetry is naturally extended to the R-parity of the MSSM.

Lastly, let us briefly discuss the issue of the discrete R-symmetry anomalies. The classification and analysis of the various possible anomalies has been discussed, for instance,

⁹ We wish to emphasize again that only *some* of the additional quadratic operators cause significant shifts of the vacuum we analyzed (and even in this case it could very well be that our qualitative conclusions remain intact), so the fact *all* of them are forbidden by the symmetries (4.14) is an added (but by no means necessary) benefit.

in [37,44,45]. In particular, [45] argued that the cubic anomaly does not impose any constraints on the low energy effective action. So, we remain with the constraints of gravitational anomaly and mixed gauge-discrete anomalies. The mixed anomaly with the strong $SU(3)$ is automatically satisfied and the mixed anomaly with $SU(N_c)$ gives

$$2N_{f1} + N_{f0} = 3r , \quad (4.17)$$

where r is some integer.

On the other hand, since we have not specified the R-charges of the SSM fields and have not even chosen the matter content,¹⁰ we are unable to calculate the gravitational anomaly and the mixed anomaly with the SSM gauge group. The specific charge assignment here should be merely considered as an example of how to implement the above mentioned idea of naturalization, and we leave a more detailed analysis to the future.

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¹⁰ In particular, to solve the $\mu/B\mu$ problem one usually introduces many new fields (e.g. [46]) and various special mutual interactions. The particular choice of the mechanism to solve $\mu/B\mu$ may affect the charge assignments of the potentially observable particles.

Appendix A. Calculating the Runaway Potential at Large Fields

For the sake of completeness, we will describe in this appendix the RGE evolution of the wavefunction of Φ_{11} for large Φ_{00} . A more general derivation of the leading-log potential for pseudo-moduli at large fields can be found in [38].

We will assume that $\langle \Phi_{00} \rangle = z \mathbb{I}_{N_{f0}}$ for simplicity; the generalization to arbitrary non-degenerate Φ_{00} will be obvious at the end. The wavefunction factors in the Kähler potential are governed by the RGEs:

$$\log \mathcal{Z}_{\Phi_{11}} = 2 \int_Q^\Lambda \frac{dQ'}{Q'} \gamma_{\Phi_{11}}(Q'; z) , \quad (\text{A.1})$$

where $\gamma_{\Phi_{11}}$ is its anomalous dimension, and Q is the RG scale. In this model, the relevant one-loop anomalous dimensions above the scale h_z are

$$\gamma_{\Phi_{11}} = \frac{Nh^2}{16\pi^2 \mathcal{Z}_{\Phi_{11}} \mathcal{Z}_{\varphi_1}^2} , \quad \gamma_{\varphi_1} = \gamma_{\widetilde{\varphi}_1} = \frac{N_{f1}h^2}{16\pi^2 \mathcal{Z}_{\Phi_{11}} \mathcal{Z}_{\varphi_1}^2} + \frac{N_{f0}h^2}{16\pi^2 \mathcal{Z}_{\Phi_{10}} \mathcal{Z}_{\varphi_0} \mathcal{Z}_{\varphi_1}} , \quad (\text{A.2})$$

and below the scale h_z they are

$$\gamma_{\Phi_{11}} = \frac{Nh^2}{16\pi^2 \mathcal{Z}_{\Phi_{11}} \mathcal{Z}_{\varphi_1}^2} , \quad \gamma_{\varphi_1} = \gamma_{\widetilde{\varphi}_1} = \frac{N_{f1}h^2}{16\pi^2 \mathcal{Z}_{\Phi_{11}} \mathcal{Z}_{\varphi_1}^2} . \quad (\text{A.3})$$

In particular, $\gamma_{\Phi_{11}}$ is continuous at the scale h_z . Therefore we have

$$\frac{\partial \log \mathcal{Z}_{\Phi_{11}}}{\partial \log h_z} = 2 \int_Q^\Lambda \frac{dQ'}{Q'} \frac{\partial \gamma_{\Phi_{11}}(Q'; z)}{\partial \log h_z} = -2 \int_Q^\Lambda \frac{dQ'}{Q'} \gamma_{\Phi_{11}} \frac{\partial (\log \mathcal{Z}_{\Phi_{11}} + 2 \log \mathcal{Z}_{\varphi_1})}{\partial \log h_z} . \quad (\text{A.4})$$

At leading-log order, only the second term in (A.4) matters, and only through the discontinuity in γ_φ :

$$\log \mathcal{Z}_{\varphi_1} = \int_{h_z}^\Lambda \frac{dQ'}{Q'} \left(\frac{N_{f1}h^2}{8\pi^2 \mathcal{Z}_{\Phi_{11}} \mathcal{Z}_{\varphi_1}^2} + \frac{N_{f0}h^2}{8\pi^2 \mathcal{Z}_{\Phi_{10}} \mathcal{Z}_{\varphi_0} \mathcal{Z}_{\varphi_1}} \right) + \int_Q^{h_z} \frac{dQ'}{Q'} \left(\frac{N_{f1}h^2}{8\pi^2 \mathcal{Z}_{\Phi_{11}} \mathcal{Z}_{\varphi_1}^2} \right) \quad (Q < h_z) , \quad (\text{A.5})$$

so

$$\frac{\partial \log \mathcal{Z}_{\varphi_1}}{\partial \log h_z} = -\frac{N_{f0}h^2}{8\pi^2 \mathcal{Z}_{\Phi_{10}} \mathcal{Z}_{\varphi_0} \mathcal{Z}_{\varphi_1}} \Theta(h_z - Q) + \dots , \quad (\text{A.6})$$

where \dots do not contribute in the leading log approximation. Substituting this into (A.4) and integrating twice, we conclude that

$$\log \mathcal{Z}_{\Phi_{11}} = \frac{Nh^2}{8\pi^2} \frac{N_{f0}h^2}{8\pi^2} \left(\log \frac{h_z}{Q} \right)^2 + \dots , \quad (\text{A.7})$$

where we have dropped the wavefunction factors since they will only contribute at higher loop order.

Now, it is trivial to generalize to arbitrary Φ_{00} . For instance, if Φ_{00} is diagonal but has different eigenvalues, the same arguments as above lead to

$$\log \mathcal{Z}_{\Phi_{11}} = \frac{Nh^2}{8\pi^2} \frac{h^2}{8\pi^2} \sum_{i=1}^{N_{f0}} \left(\log \frac{hz_i}{Q} \right)^2 + \dots . \quad (\text{A.8})$$

Since Φ_{00} can always be diagonalized by an $SU(N_{f0})_L \times SU(N_{f0})_R$ transformation, the full generalization of (A.7), (A.8) must be

$$\log \mathcal{Z}_{\Phi_{11}} = \frac{Nh^2}{16\pi^2} \frac{h^2}{16\pi^2} \text{Tr} \left(\log \frac{h^2 \Phi_{00}^\dagger \Phi_{00}}{Q^2} \right)^2 + \dots , \quad (\text{A.9})$$

which is the result quoted in the text upon substituting $Q = h\mu$.

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